

Due November 1st, 2023, 10PM Eastern

Instructions:

- Submit your assignments on Gradescope as a PDF. You may either handwrite your answers and scan them into a PDF, or type your answers and convert them to PDF. If you are handwriting your answers, please make sure your handwriting is legible.
- Clearly label any intercepts, slopes, jumps, or kinks on your graphs. If you do not label these, you will not receive full credit. Don't worry about making graphs exactly to scale; just make them reasonable.
- You only need to submit answers for graded questions. The ungraded questions are for your own edification.

1. (10 points) **Room 101**

O'Brien and the Ministry of Love seek to remake the hearts of Winston and Julia, by having them decide whether denounce each other. If neither denounces each other, both of them have a payoff of 0. If one denounces the other, the denouncer gets released, earning a payoff of 2, and the denouncee gets a payoff of 0. If both denounce each other, they both get a payoff of -2 , for selling each other out.

- (a) (2 points) Construct the payoff matrix for this game.
- (b) (2 points) What is the pure-strategy Nash equilibrium of this game?
- (c) (3 points) What is the mixed-strategy Nash equilibrium of this game?
- (d) (3 points) Suppose O'Brien promises a sinecure if **both** Winston and Julia denounce each other, making the payoff for denouncing $c - 2$. If only one person denounces, their payoff is still 2. For what values of c does a pure-strategy Nash equilibrium exist where both Winston and Julia denounce each other? What is the lowest value of c for which this is true?

2. (10 points) **Moves and Countermoves**

OpenAI and Anthropic each have a Large Language Model (LLM) they seek to sell to consumers. They compete over the number of queries they can respond to, with the market demand for queries given by:

$$P(Q) = 500 - 3Q$$

where Q is the total number of queries across both firms.

Each firm faces a constant average cost of \$2 per query.

- (a) (4 points) Suppose OpenAI can choose its output before Anthropic and that Anthropic observes OpenAI's choice before choosing their output. What is the profit-maximizing output for OpenAI?
- (b) (4 points) Suppose Anthropic and OpenAI choose to collude, sharing the profits equally. What would be their joint output?
- (c) (2 points) Calculate which of these equilibria are better for each firm. Discuss your results.

3. (10 points) **Monopsony Markets**

Gig economy companies such as DoorDash and Uber Eats can be thought of as monopsonists who can choose how much to pay gig workers. Let us narrow our focus to DoorDash in Charlottesville. Assume they are price takers in the market for food delivery and monopsonists in the market for gig workers. The market price for food delivery is \$50, and the inverse labor supply curve is $w = 10 + 10L$, where L is the number of workers hired. Furthermore, suppose $Q = 3L - 0.5L^2$ represents the number of deliveries that can be made per worker.

- (a) (7 points) What is the profit-maximizing wage, number of workers, and output (number of deliveries) for DoorDash?
- (b) (3 points) Suppose the government imposes a minimum wage of \$50. What is the new profit-maximizing level of labor and deliveries for DoorDash? How does this compare to the previous case? Discuss the changes in surplus and welfare. (Hint: the labor supply curve is now horizontal at $w = 50$.)

4. Ungraded Questions

- (a) Betty and Sammy both witness a crime. Each of them decides whether to call the police. If at least one of them calls, then the crime will be solved and, for each of them, this adds a benefit of 1 to their payoff. For both Betty and Sammy, calling the police reduces the caller's payoff by $c \in (0, 1)$. Suppose Betty and Sammy must make the decision to call or not to call as part of a static game. In other words, Betty and Sammy must make their decisions simultaneously.
- Create the payoff matrix for this game. Find all pure-strategy Nash Equilibria. Are there any strictly dominant strategies?
 - Assume $c < 0$. Find all pure-strategy Nash Equilibria.
 - Assume $c > 1$. Find all pure-strategy Nash Equilibria.
- (b) Andrew and Chloe are playing a game of hide-and-seek. Andrew is hiding and Chloe is trying to find him. Andrew can decide to hide in the Kitchen, the Living Room or the Dining room. Chloe has the time to check only one room. Assume choices are made simultaneously (each one, when deciding, doesn't know the others choice). Whoever wins receives a payoff of 1. The loser receives a payoff of -1 .
- Create the payoff matrix for this game. Why is there no pure-strategy Nash Equilibrium?
 - What will be the mixed-strategy equilibrium be in this game? You can use an intuitive explanation to justify your answer.
 - How would the equilibrium outcome(s) change if Andrew decided first and then Chloe observed Andrew's decision before making her own?