

**Due November 21st, 2023, 10PM Eastern**

**Instructions:**

- Submit your assignments on Gradescope as a PDF. You may either handwrite your answers and scan them into a PDF, or type your answers and convert them to PDF. If you are handwriting your answers, please make sure your handwriting is legible.
- Clearly label any intercepts, slopes, jumps, or kinks on your graphs. If you do not label these, you will not receive full credit. Don't worry about making graphs exactly to scale; just make them reasonable.
- You only need to submit answers for graded questions. The ungraded questions are for your own edification.

1. (5 points) **Small Island Economy**

On a certain small island, there are 100 units of labor and 200 units of capital. Two goods can be produced. Good  $A$  is produced with fixed coefficients, using 1 unit of labor and 3 units of capital per unit of output. Good  $B$  is produced with fixed coefficients, using 1 unit of labor and 1 unit of capital per unit of output. Let  $X_a$  denote the quantity of good  $A$  and  $X_b$  be the quantity of good  $B$  that is produced. The set of feasible output combinations for this economy is restricted by the fact that it cannot use more than 100 units of labor or 200 units of capital.

- (a) (2 points) Write down two inequalities expressed in terms of  $X_a$  and  $X_b$  that must be satisfied at feasible output combinations.
- (b) (3 points) Draw a graph showing the economy's production possibility set. Put numerical labels on your graph so that this graph is precisely described.

2. (10 points) **The Uses of AI**

Consider a generative AI model (e.g., GPT), which requires both compute  $C$  and data  $D$  to use. This model can do one of two things: be a chatbot (e.g., ChatGPT), or generate images (e.g., DALL-E). Each of these tasks requires a different amount of compute and data. Specifically, the production functions for each task are:

$$Q_B = \sqrt{C_B D_B}$$
$$Q_I = \sqrt{C_I D_I}$$

This economy also has two types of consumers: chatbot users and image users. Each type of consumer has a different utility function, and each type of consumer values the two goods differently. Specifically, the utility functions are:

$$U_G = Q_B^{3/4} Q_I^{1/4}$$
$$U_M = Q_B^{1/4} Q_I^{3/4}$$

Each consumer can produce one unit of data and one unit of compute, and there are 150 consumers of each type.

- (a) (5 points) Solve for the production contract curve  $D_B(C_B)$  and derive the transformation schedule (i.e., the production possibility curve) for this economy.
- (b) (5 points) Using the utility for each type of consumer and the production possibility curve from part (a), find the conditions (equations) for the set of Pareto optimal allocations of chatbots and images in this economy.

### 3. Ungraded Questions

Consider a generative AI model (e.g., GPT), which requires data  $C$  to use. This model can do one of two things: be a chatbot (e.g., ChatGPT), or generate images (e.g., DALL-E). Each of these tasks requires a different amount of compute and data. Specifically, the production functions for each task are:

$$B = \sqrt{C_B}$$
$$I = 0.5\sqrt{C_I}$$

Each consumer can produce one unit of compute, and there are 300 consumers in total. Solve for the production possibility curve in this economy.