

Due November 8th, 2023, 10PM Eastern

Instructions:

- Submit your assignments on Gradescope as a PDF. You may either handwrite your answers and scan them into a PDF, or type your answers and convert them to PDF. If you are handwriting your answers, please make sure your handwriting is legible.
- Clearly label any intercepts, slopes, jumps, or kinks on your graphs. If you do not label these, you will not receive full credit. Don't worry about making graphs exactly to scale; just make them reasonable.
- You only need to submit answers for graded questions. The ungraded questions are for your own edification.

1. (15 points) **Shipping Wars**

UPS and FedEx are the two largest shipping providers in the US. Suppose that they make up a symmetric duopoly, and face an inverse market demand curve of $P = 200 - 4Q$, where Q is the total quantity of packages shipped by both firms. Each firm has a constant marginal cost of \$20 per package, and no fixed cost.

- (a) (2 points) Suppose UPS and FedEx compete on quantity. Find their reaction functions and draw them on a graph.
- (b) (2 points) What is the Cournot equilibrium (price and quantities) for this market?
- (c) (2 points) Suppose instead these firms compete on price. What is the Bertrand equilibrium (price and quantities) for this market?
- (d) (2 points) Which equilibrium is best for consumers? for UPS? for FedEx? Explain.
- (e) (7 points) Suppose that UPS and FedEx produce complementary goods, rather than substitute goods. For example, UPS warehouses packages and ships them to a FedEx location, and FedEx handles the final delivery to the customer. In this example, demand for UPS and FedEx are the following:

$$Q_U = 200 - 2P_U - 2P_F$$

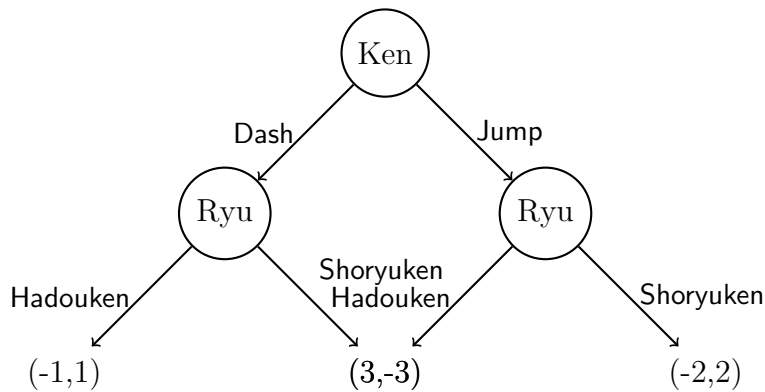
$$Q_F = 200 - 2P_U - 2P_F$$

Marginal costs are still \$20 per package, and there are no fixed costs. Assume UPS and FedEx compete on **price**.

- i. (2 points) Derive UPS's and FedEx's reaction functions in terms of prices (rather than quantities).
- ii. (3 points) Solve for the Bertrand equilibrium (prices and quantities) in this market.
- iii. (2 points) How does this equilibrium compare to the Bertrand equilibrium in part (c)? Explain the economic intuition behind the difference (or lack thereof) in the equilibria.

2. (5 points) **Fighting Game Theory**

Ryu and Ken are in a street fight. Ken can move before Ryu, and can choose to dash or jump. In response to Ken's move, Ryu can choose to use a Hadouken, countering the dash or a Shoryuken, countering the jump. If Ryu responds correctly with a Hadouken, payoffs are $(-1, 1)$ for Ken and Ryu, respectively. If Ryu responds correctly with a Shoryuken, payoffs are $(-2, 2)$ for Ken and Ryu, respectively. If Ryu responds incorrectly, payoffs are $(3, -3)$ for Ken and Ryu, respectively. We can write this game in **extensive form** as follows:



- (a) (2 points) What is the subgame perfect Nash equilibrium of this game?
- (b) (3 points) Suppose Ryu and Ken must choose simultaneously, rather than sequentially (payoffs are the same). What is the mixed strategy Nash equilibrium of this game?

3. Ungraded Questions

- (a) Consider a market with three firms producing the same homogeneous good. The inverse demand is $P = e^{-Q}$, where P is the price and Q is the quantity. The three firms have zero costs of production.
- Find the Cournot equilibrium of this market.
 - Find the set of Bertrand equilibria of this market.
 - Find the Stackelberg equilibrium of this market, assuming firm 3 follows firm 2, who follows firm 1.
- (b) Suppose that there are only two firms in a market, 1 and 2, which both face a demand curve of $D = 10 - p$ for their product. After many years of fighting in quantity competition, the executives of firms 1 and 2 meet in a smoke-filled room and decide to collude. That is, they solve their profit maximization problem jointly. Assume that if they decide on a total quantity Q , each firm is supposed to make half of it. Let each firm's cost function be given by $C(q_i) = 2q_i$ for $i = 1, 2$. After the decision is made, the executives go back to their plant and announce how much quantity they will each produce.
- Write down the profit maximization problem that the firms solve together.
 - Solve the problem in (a) for the optimal values of $Q, q_1, q_2, \pi_1, \pi_2, p$, where π_i is firm i 's profit.

For the rest of the problem, suppose that the firms choose how much to produce in each period $t = 0, 1, 2, \dots$. Suppose also that after production, each firm knows exactly how much the other firm produced. Suppose further that if one firm produces anything other than the cartel quantity, all firms will afterward produce the perfectly competitive amount, resulting in 0 profits. Finally, suppose that each firm has a discount factor, δ .

- Find what the perfectly competitive equilibrium would be (prices, quantities, profits).
- Find what a firm would produce if it were to cheat, and the profit it would make. You can take it as given that the other firm would play good and produce the cartel quantity it is assigned.
- How high does the discount factor need to be for neither firm to ever want to cheat?