

Due September 20th, 2023, 10PM Eastern

Instructions:

- Submit your assignments on Gradescope as a PDF. You may either handwrite your answers and scan them into a PDF, or type your answers and convert them to PDF. If you are handwriting your answers, please make sure your handwriting is legible.
- Clearly label any intercepts, slopes, jumps, or kinks on your graphs. If you do not label these, you will not receive full credit. Don't worry about making graphs exactly to scale; just make them reasonable.
- You only need to submit answers for graded questions. The ungraded questions are for your own edification.

1. (10 points) **Movie Madness, Part 2: Slutsky's Revenge!**

Lars is a movie aficionado, and he exchanges pirated movies on a file-sharing service. He has a collection of 7 movies pirated from (HBO) Max, and 6 pirated movies from Netflix. His utility over the number of movies he pirates is $U(m, n) = m^{\frac{4}{5}} n^{\frac{1}{5}}$.

- (a) (4 points) On this file-sharing service, the exchange value of one Max movie is two Netflix movies (i.e., a user can trade one Max movie to receive two Netflix movies, or vice-versa). What is Lars's optimal collection of pirated movies? Is he a net uploader or downloader of Netflix and Max movies?
- (b) (3 points) Write Lars's net demand function for Max movies as a function of movie exchange value v_m and endowment ω_m . Graph this demand curve, with quantity on the horizontal axis and exchange value on the vertical axis. Marking his initial endowment.
- (c) (3 points) Suppose the exchange value of Netflix movies increases, such that Max and Netflix movies have equal value (i.e., a user can exchange one Netflix movie for one Max movie, or vice-versa). What is Lars's new optimal allocation of movies between Max and Netflix? Decompose the change in Lars's demand for Netflix movies into the substitution effect, the income effect, and the endowment effect.

2. (10 points) A Consumption-Savings Model

Lucille is considering how to allocate her consumption for the next two years. She had the following utility function over consumption:

$$U(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$$

where $\beta \in (0, 1)$ represents Lucille's discount factor for year 2 consumption. After two years, her discount factor is zero, so she has no utility from consumption after the second year. The price of consumption in the first year is \$1 and the price of consumption in the second year is p_2 . She can also borrow and lend at an interest rate of r . She receives y income in the first year and y income in the second year.

- (a) (2 points) Write down Lucille's intertemporal budget constraint.
- (b) (2 points) Write down Lucille's optimization problem (a Lagrangian is not necessary here, but it may be helpful).
- (c) (2 points) Derive an optimality condition that relates consumption in the first year to consumption in the second year. What is the economic intuition behind this condition?
- (d) (2 points) Using your derivations from part (c), solve for the optimal consumption in each year as a function of y , β , r , and p_2 .
- (e) (2 points) Suppose you are a policymaker at the Federal Reserve. Using your answer from part (d), how would you change the interest rate to encourage Lucille to save more? What is the economic intuition behind your answer?

3. (10 points) **Ministry for Present and Future Values**

- (a) (5 points) Suppose you own a tree. The size of the tree each year is given by $S(T) = \sqrt{T}$, where T is the number of years since you planted the tree. The price of lumber is $p = \$1$ per meter, and the interest rate is $r = 0.25$. What is the optimal time to cut down the tree? Explain your reasoning.
- (b) (5 points) Consider the market for car batteries, which has two perfectly substitutable goods: lithium, a depletable resource, and magnesium, a renewable resource. Global demand for car batteries is 1 million batteries per year, and there are only 10 million batteries worth of lithium in the ground. Assume zero cost to extracting lithium or producing lithium batteries, but that the cost of producing a magnesium battery is $c = \$100000$ per battery. The annual interest rate is $r = 0.05$. What is the equilibrium price of lithium batteries? Explain your reasoning.

4. Ungraded Questions

- (a) Suppose there are 2 periods $t = 1, 2$. In period t , consumption is c_t . Income y is the same for both periods. The consumer's utility function is $U(c_1, c_2) = c_1^\alpha c_2^{1-\alpha}$, $\alpha \in (0, 1)$.
- Suppose $y = 10$, and no financial market exists. Find out the consumer's optimal bundle of (c_1, c_2) for $\alpha = 0.25$, $\alpha = 0.5$, and $\alpha = 0.75$.
 - Suppose $y = 10$, and the consumer can borrow and save at an interest rate of r . For what range of α will the consumer borrow in period 1, and for what range of α will the consumer save in period 1?
- (b) Andy and Betty are both asked to make a series of decisions. Andy's utility of wealth function is $U(x) = \ln(x)$ and Betty's utility of wealth function is $U(x) = e^x$. They both aim to maximize their expected utility.
- Andy and Betty are each offered either \$10 guaranteed or \$5 with .5 probability and \$15 with .5 probability. What option will Andy choose? What option will Betty choose? Without using numbers, justify your answers. (Graphical arguments or general economic arguments are acceptable here)
 - Andy and Betty both still have the option to pick the offer of \$5 with .5 probability and \$15 with .5 probability. However, their other option is now \$0 with .5 probability and \$20 with .5 probability. What option will Andy choose? What option will Betty choose? (Graphical arguments or general economic arguments are acceptable here)
 - Andy and Betty are each offered either \$12 guaranteed or \$5 with .5 probability and \$15 with .5 probability. What options will Andy and Betty choose? Explain
- (c) Nick consumes chocolate over two periods. He has 20 chocolate bars which can be consumed in either period. He cannot buy more chocolate bars and left over chocolate bars do not gain or lose value. Let c_1 be the amount of chocolate bars consumed in period 1 and let c_2 be the amount of chocolate bars consumed in period 2.

Unfortunately for Nick, there is a .25 probability that someone will steal his chocolate before he ever gets a chance to eat it. Ian the insurance broker offers to replace any stolen chocolate as long as Nick pays Ian F upfront for the insurance. Nick's utility is $U(c_1; c_2; F') = c_1 c_2 - F'$ where c_1 and c_2 are the actual amounts of chocolate consumed and F' is the amount spent on insurance (0 if no insurance is purchased, F if insurance is purchased). Nick maximizes his expected utility.

Find the threshold price F^* for insurance where Nick is indifferent over buying insurance. What happens if $F > F^*$? What happens if $F < F^*$?