

Due October 18th, 2023, 10PM Eastern

Instructions:

- Submit your assignments on Gradescope as a PDF. You may either handwrite your answers and scan them into a PDF, or type your answers and convert them to PDF. If you are handwriting your answers, please make sure your handwriting is legible.
- Clearly label any intercepts, slopes, jumps, or kinks on your graphs. If you do not label these, you will not receive full credit. Don't worry about making graphs exactly to scale; just make them reasonable.
- You only need to submit answers for graded questions. The ungraded questions are for your own edification.

1. (10 points) **Spinning a Thread**

Suppose you are launching a new social media platform. Your goal is to maximize the profit of selling users to advertisers. To acquire users, you require two inputs: influencers, and server capacity. Specifically, your production function for users is:

$$U = 10\min\{I, 10S\}$$

U is the number of users, I is the number of influencers, and S is the amount of server capacity. Let the price of influencers be p_I and the price of server capacity be p_S . p_U is the price at which you can sell users to advertisers.

- (a) (2 points) Write the firm's profit maximization problem.
- (b) (2 points) Suppose that server capacity is fixed in the short run at $S = 10$. Let $p_I = 10$, $p_S = 20$, and $p_U = 1$. What is the profit-maximizing number of influencers?
- (c) (2 points) Suppose server capacity is still fixed at $S = 10$, but now $p_I = 15$, $p_S = 20$, and $p_U = 0.5$. What is the profit-maximizing number of influencers?
- (d) (4 points) Suppose we are in the long run, and the firm can choose any level of server capacity. Write the cost minimization problem, find the conditional factor demands $I(p_I, p_S, U)$ and $S(p_I, p_S, U)$, and find the cost function $C(p_I, p_S, U)$

2. (10 points) **Ice Cream Machines**

Consider the economy for ice cream. An ice cream shop has two machines to make ice cream. The cost function for each machine is given by:

$$C_1(q_1) = 18 - 10q_1 + 2q_1^2$$

$$C_2(q_2) = 32 - 6q_2 + 3q_2^2$$

where q_1 is the quantity produced on machine 1 and q_2 is the quantity produced on machine 1.

- (a) (3 points) Find the marginal cost, average total cost, average variable cost, and average fixed cost of each machine.
- (b) (2 points) Graph the average total cost, average fixed cost, and marginal cost for machine 1.
- (c) (5 points) Suppose the shop wants to produce 20 units of ice cream. How many units should it produce on each machine?

3. (10 points) Mapping Production to Cost

Match each of the following production functions with the corresponding total cost function. Show your work; just matching will not gain you many points. You may assume that all values are non-negative. Note that some production functions may have the same cost function, so I discourage against working by elimination.

(a) (2.5 points) $F(L, K) = (L + K)^2$

(b) (2.5 points) $F(L, K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$

(c) (2.5 points) $F(L, K) = L^2 + K^2$

(d) (2.5 points) $F(L, K) = \min\{L, K\}^2$

The possible cost functions are given by:

1. $C(q, w, r) = 2q\sqrt{rw}$

2. $C(q, w, r) = \sqrt{qwr}$

3. $C(q, w, r) = \min\{w, r\}\sqrt{q}$

4. $C(q, w, r) = (r + w)\sqrt{q}$

4. Ungraded Questions

- (a) Consider the economy for ancient Greek textbooks. The cost function for a textbook manufacturer is $C(q) = 2q^2 - 4q + 18$.
- Write the average total cost, average variable cost, and marginal cost functions for the textbook manufacturer.
 - Suppose we are in the short-run, and the market price for textbooks is \$6. What is the short-run profit-maximizing level of output for the textbook manufacturer? Does the firm supply the market at this price?
 - Suppose we are in the long run with free entry. What is the efficient level of output for the textbook manufacturer? What must the price be for the firm to remain in the market?
 - Suppose the market demand function is $Q = 32 - P$. Again assuming free entry, what is the long-run number of firms in the market, based on your results from part (c)? Assume each textbook manufacturer has the same cost function.
- (b) Suppose you have a large language model (think ChatGPT). This large language model requires two inputs to function: compute (computing power) and data. The production function for this LLM is given by $Y = AC^{\frac{3}{5}}D^{\frac{1}{5}}$, where Y is the output of the LLM (responses to queries), A is a constant, C is the amount of compute, and D is the amount of data. p_Y is the price per unit of output, p_C is the price per unit of compute, and p_D is the price per unit of data.
- Sketch the Isoquants for this production function (assume $A = 1$ exclusively for this part)
 - Does this production function exhibit constant, increasing, or decreasing returns to scale? Explain.
 - Solve this firm's long-run profit maximization problem. Specifically, find the supply function $Y(p_C, p_D, p_Y)$.
 - Solve the long-run cost minimization problem. Find the conditional factor demands for data and compute $(D(p_d, p_c, Y), C(p_d, p_c, Y))$, and the associated cost function $C(p_c, p_d, Y)$.
 - Suppose the CEOs of major social platforms limit the amount of scraping modelers can do, doubling the price of data. Using your cost function from part (d), determine how much more expensive the LLM becomes for the same level of output.