

**Due September 13th, 2023, 10PM Eastern**

**Instructions:**

- Submit your assignments on Gradescope as a PDF. You may either handwrite your answers and scan them into a PDF, or type your answers and convert them to PDF. If you are handwriting your answers, please make sure your handwriting is legible.
- Clearly label any intercepts, slopes, jumps, or kinks on your graphs. If you do not label these, you will not receive full credit. Don't worry about making graphs exactly to scale; just make them reasonable.
- You only need to submit answers for graded questions. The ungraded questions are for your own edification.

1. (6 points) **Movie Mindset**

Consider two films, currently scheduled to be released on the same day: a concert film by an internationally beloved artist, and the latest entry in a long-running horror franchise. The horror studio is reconsidering its film's release date. The studio has several models for consumer demand for the two films. These models are the following sets of equations:

$$\text{Model 1: } \begin{cases} x_c &= 10000 - 2p_c + p_h \\ x_h &= 100 - 2p_h + p_c \end{cases}$$

$$\text{Model 2: } \begin{cases} x_c &= \frac{w}{3p_h + p_c} \\ x_h &= \frac{3w}{3p_h + p_c} \end{cases}$$

$$\text{Model 3: } \begin{cases} x_c &= \frac{9}{10} \frac{w}{p_c} \\ x_h &= \frac{1}{10} \frac{w - 2w^2}{p_h} \end{cases}$$

Where  $x_c$  and  $x_h$  are the number of tickets sold for the concert film and the horror film, respectively,  $p_c$  and  $p_h$  are the prices of the concert film and the horror film, respectively, and  $w$  is the total wealth of consumers.

- (a) (3 points) For each of the above models, answer the following questions. Justify your solutions mathematically.
- (1 point) Are the movies normal, income inferior, or neither?
  - (1 point) Are the movies substitutes, complements, or neither?
  - (1 point) Are the movies ordinary or Giffen?
- (b) (3 points) Suppose the horror studio has hired you to advise them on whether to move the release date of their film. What advice would you give them for each of the above models? Explain your reasoning.

2. (12 points) **Movie Madness**

Greg is a movie aficionado, and he watches movies on streaming services. He currently subscribes to (HBO) Max and Netflix, and his utility over the number of movies watched on each service is  $U(m, n) = m^{\frac{4}{5}}n^{\frac{1}{5}}$ . Greg has 20 hours of leisure over the course of a week to spend watching movies. Movies on Max are 2 hours long, and movies on Netflix are 1 hour long.

- (a) (4 points) What is Greg's optimal allocation of movies between Max and Netflix? How many hours does he spend watching movies on each service?
- (b) (4 points) Would any of the following utility functions change Greg's optimal allocation? Explain your reasoning.
  - i. (1 point)  $U(m, n) = m^{32}n^8 + 200$
  - ii. (1 point)  $U(m, n) = (-m)^{32}n^8$
  - iii. (1 point)  $U(m, n) = m^{\frac{1}{2}}n^{\frac{1}{2}}$
  - iv. (1 point)  $U(m, n) = m^{\frac{4}{5}} + n^{\frac{1}{5}}$
- (c) (4 points) Suppose the CEO of Max requires that Greg watch 30 minutes of advertisements for every movie he watches on Max after the first two movies. What is Greg's new optimal choice of movies? How much time does he spend watching advertisements? (Hint: you should write out Greg's new budget constraint)

**3. (12 points) Continental Breakfast**

A Continental Breakfast usually consists of pastries and coffee (or tea). The latest style guides from Paris recommend that the ratio of pastries to cups of coffee should in any Continental Breakfast should be 2:1.

The price of a pastry is \$2, and the price of a cup of coffee is \$1, and consumers have a budget of \$20.

- (a) (4 points) Write and solve the consumer's optimization problem over pastries and coffee. What is the consumer's optimal bundle of pastries and coffee?
- (b) (2 points) Write the consumer's demand function for pastries as a function of the price of pastries  $p_s$ , the price of coffee  $p_c$ , and the consumer's budget  $w$ . Graph the demand curve for pastries.
- (c) (3 points) Sensing a business opportunity, cafés in Paris begin offering a "Continental Breakfast" bundle, consisting of two pastries and a cup of coffee for \$4. How many "Continental Breakfast" bundles will the consumer purchase?
- (d) (3 points) Can you find a condition on the price of pastries  $p_s$  and the price of coffee  $p_c$  such that the consumer would always purchase them separately, rather than as part of a "Continental Breakfast" bundle? If so, provide the condition. If not, explain why not.

#### 4. Ungraded Questions

(a) Derive the demand curve for the following utility functions:

- $U(x, y) = ax + by$
- $U(x, y) = \min\{ax, by\}$
- $U(x, y) = x^a y^b$
- $U(x, y) = a \ln(x) + y$
- $U(x, y) = (ax^r + bx^r)^{\frac{1}{r}}$

(b) Suppose you are shopping for cereal and have a budget of \$20. Cheerios cost \$3 per box, and Lucky Charms cost \$6 per box. Your utility over Cheerios and Lucky Charms is given by  $U(c, l) = c^{\frac{1}{5}} l^{\frac{4}{5}}$ , where  $c$  is the number of boxes of Cheerios you buy, and  $l$  is the number of boxes of Lucky Charms you buy.

- i. What is the marginal rate of substitution of Cheerios for Lucky Charms?
- ii. What is your optimal bundle of cereal?
- iii. Sketch your indifference curve, budget line at the optimal bundle of cereal. Identify the optimal bundle on your graph.
- iv. Suppose the price of Lucky Charms increases to \$8 per box. On your graph from part (c), show how this affects your optimal bundle of cereal, highlighting both the income and substitution effects.

(c) Sven, Andrew and Becky each have an income of \$5. Initially the price of commodity x and the price of commodity y are both \$1. The price of commodity x increases from \$1 to \$3.

- i. Sven's utility function is  $U(x, y) = \min(3x, 4y)$ . Graph the income and substitution effect for Sven resulting from the price increase.
- ii. Andrew's utility function is  $U(x, y) = 4x + 3y$ . Graph the income and substitution effect for Andrew resulting from the price increase.
- iii. Becky's utility function is  $U(x, y) = x^{\frac{2}{3}} y^{\frac{1}{3}}$ . Graph the income and substitution effect for Becky resulting from the price increase.