

**Due September 27th, 2023, 10PM Eastern**

**Instructions:**

- Submit your assignments on Gradescope as a PDF. You may either handwrite your answers and scan them into a PDF, or type your answers and convert them to PDF. If you are handwriting your answers, please make sure your handwriting is legible.
- Clearly label any intercepts, slopes, jumps, or kinks on your graphs. If you do not label these, you will not receive full credit. Don't worry about making graphs exactly to scale; just make them reasonable.
- You only need to submit answers for graded questions. The ungraded questions are for your own edification.

1. (10 points) **E[To The Moon!]**

Consider an investor with the following utility function over wealth:

$$U(w) = \begin{cases} -\frac{e^{-\alpha w}}{\alpha} & \text{if } \alpha \neq 0 \\ w & \text{if } \alpha = 0 \end{cases}$$

He is considering a financial asset that has the following payoff structure:

State	Probability	Payoff
1	0.1	\$100
2	0.9	\$0

Alternatively, he can invest in a risk-free asset that pays \$10 regardless of the state of the world. Assume that the investor cannot diversify his portfolio and invest in both assets. He can choose only one.

- (a) (2 points) For what values of  $\alpha$  will the investor invest in the risky asset? For what values of  $\alpha$  will the investor invest in the risk-free asset? For what values of  $\alpha$  will the investor be indifferent between the two assets? Explain your reasoning.
- (b) (4 points) Suppose  $\alpha = -1$ . What would the probability of the states of the world have to be for the investor to be indifferent between the two assets? What is the economic intuition for the magnitude (or lack thereof) of this probability? (Hint: start with the expected utility for each asset.)
- (c) (4 points) Suppose  $\alpha = 1$ . How much would the investor be willing to pay (or be compensated) to purchase the risky asset, rather than invest in the risk-free asset? (Hint: how much more or less money would the investor need to be indifferent between the two assets?)

**2. (10 points) Compensating Corner Juice**

Consider the market for smoothies and açai bowls. A consumer has the following utility function over smoothies and bowls:

$$U(s, a) = s^{\frac{7}{10}} a^{\frac{3}{10}}$$

- (a) (2 points) Suppose consumers face equilibrium prices \$5 and \$7.5 for smoothies and bowls, respectively. They have \$50 in wealth. Solve the consumer's optimization problem. Provide both their demand function (as functions of  $p_s$ ,  $p_a$ , and  $w$ ) and their optimal quantities of smoothies and bowls.
- (b) (4 points) Suppose the price of smoothies increases to \$7. What is the compensating variation, equivalent variation, and the change in consumer surplus (all compared to part (b))?
- (c) (2 points) Explain the differences (or lack thereof) between these three measures.
- (d) (2 points) How would your answer to part (c) change if utility were  $U(s, a) = s^{\frac{7}{10}} + a$ ? You do not need to re-solve the consumer's optimization problem, an explanation will suffice.

3. (10 points) **Temu, TikTok, and Taxes**

Consider the market for imported phone cases (no domestic case manufacturers exist). Consumers can purchase these cases from two different apps: Temu and TikTok Shopping. The demand curves for cases on these two apps are given by the following equations:

$$D_T = 30 - 2P$$

$$D_S = 60 - P$$

where  $T$  is Temu and  $S$  is TikTok Shopping.

The aggregate supply curve for cases is given by the following equation:

$$S_C = 30 + 2P$$

Suppose there are many case manufacturers and TikTok Shopping and Temu simply pass through the price of cases, such that there exists a competitive equilibrium.

- (a) (2 points) Construct the aggregate demand curve for phone cases.
- (b) (2 points) Find the equilibrium price and quantity of phone cases.
- (c) (2 points) Find the own-price elasticity of demand at the equilibrium price and quantity.
- (d) (2 points) Suppose the government places a per unit tax of \$6. Find the new equilibrium prices for consumers and producers and the new equilibrium quantity.
- (e) (2 points) What is the incidence of the import tax in part (d)? How much of the tax is borne by consumers? How much is borne by producers?
- (f) (0 points (bonus)) Suppose Temu and TikTok Shopping competed directly for consumers and could set prices, taking the price the case manufacturers charge as a cost. Discuss how that might affect the equilibrium price and quantity of cases.

**4. Ungraded Questions**

- (a) A consumer's utility function is  $U = 2\sqrt{a} + b$ ,  $p_a = 1$ ,  $p_b = 1$ , and income is 10.
- Derive the demand curve for  $a$
  - Suppose  $p_a$  increases to 2. Find the compensating and equivalent variation.
  - Now the utility function is  $U = \sqrt{ab}$ . As above,  $p_a = 1$ ,  $p_b = 1$ , and income is 10. If  $p_a$  increases to 2, find the compensating variation, equivalent variation, and change in consumer surplus.
- (b) Suppose demand for frozen yogurt is given by  $P = 10 - 2Q$
- What is the consumer surplus at a price of \$4? At a price of \$7?
  - What is the price elasticity of demand at a price of \$4? At a price of \$7?
  - Now suppose that the frozen yogurt producer enters a new city, with demand  $P = 20 - Q$ . What is the aggregate demand curve?
- (c) Consider the market for model trains. This market is perfectly competitive, with the following supply and demand curves:

$$Q_D = 40 - 2P$$

$$Q_S = 10 + 3P$$

- What is the equilibrium price and quantity in this market?
- What are the elasticities of supply and demand at the equilibrium price and quantity?
- Now suppose that the government provides a \$2 per unit subsidy on model trains. What is the new equilibrium price and quantity?