

**Due November 15th, 2023, 10PM Eastern**

**Instructions:**

- Submit your assignments on Gradescope as a PDF. You may either handwrite your answers and scan them into a PDF, or type your answers and convert them to PDF. If you are handwriting your answers, please make sure your handwriting is legible.
- Clearly label any intercepts, slopes, jumps, or kinks on your graphs. If you do not label these, you will not receive full credit. Don't worry about making graphs exactly to scale; just make them reasonable.
- You only need to submit answers for graded questions. The ungraded questions are for your own edification.

1. (18 points) **Trading Card Games**

Yael and Kevin play a trading card game. This card game has two kinds of cards: monster cards, and spell cards. Kevin prefers monster cards to spell cards. His utility is  $U(m_k, s_k) = m_k^{\frac{3}{4}} s_k^{\frac{1}{4}}$ , where  $m$  is the number of monster cards and  $s$  is the number of spell cards. Yael prefers spell cards; his preferences for cards are  $U(m_y, s_y) = m_y^{\frac{1}{4}} s_y^{\frac{3}{4}}$ . Both of them buy some booster packs. Yael gets 24 monster cards and 16 spell cards from his boosters, making his endowment  $(24, 16)$ , and Kevin gets 16 monster cards and 24 spell cards from his boosters, making his endowment  $(16, 24)$ . Yael and Kevin are both willing to trade with each other.

- (a) (3 points) Draw an Edgeworth box for this economy with Yael on the lower left and Kevin on the upper right. Place monster cards on the horizontal axis and spell cards on the vertical axis. Label the axes and the endowment point.
- (b) (5 points) Derive the contract curve  $m_y(s_y)$  algebraically.
- (c) (5 points) Fix the price of spell cards to \$1. What are the Walrasian equilibrium price of monster cards and quantities of each type of card. Is it Pareto efficient?
- (d) (5 points) A Pareto efficient allocation between Yael and Kevin is  $m_y = 4, s_y = 20, m_k = 36, s_k = 20$ . Again normalizing the price of spell cards to \$1, what is the Walrasian equilibrium price of monster cards? (Hint: What must the price ratio be equal to in equilibrium?)

**2. (6 points) Verifying Veracity**

State whether each of the following is true, false or uncertain. If it is true, prove it. If false, explain why or give a counterexample. If uncertain, explain what is at issue. Assume that all questions are in reference to Edgeworth boxes (2 goods, 2 consumers), and that both consumers have weakly monotone utility functions.

- (a) (2 points) For any fixed endowment, all Pareto efficient points are competitive equilibria.
- (b) (2 points) With Cobb Douglas preferences for both consumers, the contract curve is a straight line. Hint: Consumers may have different preferences as long as both preferences are Cobb Douglas.
- (c) (2 points) The origins on the Edgeworth Box are Pareto Efficient.

3. (6 points) **A Tempestuous Exchange**

A small island economy has only two consumers, Prospero and Caliban. This economy has two goods, food  $f$  and money  $m$ .

Prospero's utility function is  $U(m, f) = m + 154f^{1/2}$ . Caliban's utility function is  $U(m, f) = m + 7f$ . At a Pareto optimal allocation in which both individuals consume some of each good, how much food does Prospero consume? (Note: Assume an interior solution exists for both Prospero and Caliban, and normalize the price of money to 1.)

#### 4. Ungraded Questions

- (a) Consider an economy with two individuals, Josh and Emma. There are two goods,  $x$  and  $y$ , the quantities of both are 4. Josh and Emma have the same Cobb-Douglas preferences, described by the utility function  $u(x, y) = \ln x + \ln y$ .
- Solve for the contract curve and illustrate your solution with a graph.
  - Suppose Josh and Emma have fixed initial endowments of  $x$  and  $y$ :  $(0, 4)$  for Josh and  $(4, 0)$  for Emma, so the total endowment is  $(4, 4)$ . What is the price ratio in the competitive equilibrium?
- (b) Suppose that James Buchanan (the economist, not the president) met Friedrich Hayek at a conference. Hayek brought with him one kilo of deep-dish pizza (D), and Buchanan brought one kilo of Virginia tobacco (V). Their respective utility functions were :  $U_h = D_h + 3V_h$  and  $U_b = 3D_b + V_b$ .
- Find the Pareto optimal set (a verbal description of the set is sufficient) and determine the competitive equilibrium allocation (this means you have to find the prices). Provide a graph that illustrates the Pareto optimal set and the competitive equilibrium allocation.
  - Repeat (a) with a new allocation in which Hayek has  $1/3$  of a kilo of pizza, and Buchanan has a kilo of Virginia tobacco plus a kilo of pizza. Provide a graph that illustrates the Pareto optimal set and the competitive equilibrium allocation.