

Due October 11th, 2023, 10PM Eastern

Instructions:

- Submit your assignments on Gradescope as a PDF. You may either handwrite your answers and scan them into a PDF, or type your answers and convert them to PDF. If you are handwriting your answers, please make sure your handwriting is legible.
- Clearly label any intercepts, slopes, jumps, or kinks on your graphs. If you do not label these, you will not receive full credit. Don't worry about making graphs exactly to scale; just make them reasonable.
- You only need to submit answers for graded questions. The ungraded questions are for your own edification.

1. (10 points) **Making Music**

A record studio produces songs q using artists A and instruments I . At first, the production function is $q = f(A, I)$. Then, the studio updates its technology, and is able to produce 30% more songs for every pair of inputs A and I .

- (a) (2 points) Write the new production function in terms of the original production function f .
- (b) (1 point) Do the marginal products of artists and instruments change? Explain.
- (c) (1 point) Does the marginal rate of technical substitution change? Explain.
- (d) (4 points) Suppose the studio doesn't update its technology, but each artist they hire is now more productive thanks to Generative AI. As a result, it can produce the same number of songs with the new bundle $(\frac{1}{3}A, I)$ as it could with the old bundle (A, I) before the change. Write down the new production function in terms of the original production function f .
- (e) (2 points) Do the marginal products and marginal rate of technical substitution change? Explain.

2. (5 points) **Textbook Technology**

Suppose we have a production technology for textbooks, which takes three inputs: ink, paper, and editors. This production technology has the following functional form:

$$Y(I, P, E) = I^{1/4} P^{1/4} E^{1/2}$$

- (a) (1 point) What are the marginal products of each input?
- (b) (1 point) Does this production technology exhibit constant, increasing, or decreasing returns to scale?
- (c) (3 points) Suppose that a materials shortage makes ink and paper fixed in the short run at $I = 81$ and $P = 256$. Treat the market price for textbooks as $P_t = 50$. Write the short-run profit maximization problem for a textbook producer. Treat the input costs for paper and ink as $w_p = 1$ and $w_I = 5$. What is the short-run demand for editors as a function of their wage w_e ?

3. Ungraded Questions

- (a) Prove whether the following production functions exhibit constant, increasing, or decreasing returns to scale:
- i. $F(K, L) = (K^2 + L^2)^{\frac{1}{2}}$
 - ii. $F(K, L) = K^{\frac{1}{2}} + L^{\frac{1}{2}}$
 - iii. $F(K, L) = \min\{L^2, K^2\}$
 - iv. $F(K, L) = L + K^2$
 - v. $F(K, L) = (L + K)^3$
- (b) For each of the above production functions, do the following:
- i. Assume $K = 1$. Write the Average and Marginal productivities of labor as a function of L .
 - ii. Write the MRTS (Marginal Rate of Technical Substitution) as a function of L and K (do not assume $K = 1$).