

**Due December 5th, 2023, 10PM Eastern**

**Instructions:**

- Submit your assignments on Gradescope as a PDF. You may either handwrite your answers and scan them into a PDF, or type your answers and convert them to PDF. If you are handwriting your answers, please make sure your handwriting is legible.
- Clearly label any intercepts, slopes, jumps, or kinks on your graphs. If you do not label these, you will not receive full credit. Don't worry about making graphs exactly to scale; just make them reasonable.
- You only need to submit answers for graded questions. The ungraded questions are for your own edification.

1. (10 points) **A Theory of Justice and Monsters**

Consider an economy with a benevolent social planner and two agents. This economy has one good,  $C$ , of which there are 18 units.

The agents have the following utility functions:

$$U_1(C_1) = C_1^\alpha$$

$$U_2(C_2) = C_2^\beta$$

Note: some of your answers may require using the quadratic formula. You may leave your answers in this form, but provide a unique solution for full credit.

- (a) (2 points) Suppose  $\alpha = \beta = 0.5$ . What is the socially optimal allocation of  $C$  for a Benthamite social planner who places equal weight on the welfare of each agent? What is the socially optimal allocation of  $C$  for a Rawlsian social planners?
- (b) (2 points) Suppose  $\alpha = \beta = 1$ . Solve for the allocations as you did in part (a).
- (c) (2 points) Suppose  $\alpha = 0.5, \beta = 1$ . Solve for the allocations as you did in part (a).
- (d) (2 points) Suppose  $\alpha = 1, \beta = 2$ . Solve for the allocations as you did in part (a).
- (e) (2 points) Discuss the differences between your results in the above parts, commenting on the economic intuition and focusing on the differences in allocations across parts.

**2. (5 points) AI as a Public Good**

Sam has a technology to build a Generative AI model (think GPT or Stable Diffusion), which we shall consider a public good. This economy has two types of consumers: 75 researchers and 200 creatives. Researchers and creatives have the following utility functions:

$$U_R(q, w_r) = 3\ln(q) + w_r$$

$$U_C(q, w_c) = 2\ln(q) + w_c$$

where  $q$  is the quality of the model and  $w$  is their wealth. Suppose the cost of building the model is  $C(q) = 25q$ .

- (a) (5 points) What is the socially optimal quality of the model?
- (b) (5 points (bonus)) Assume that the economy only has one researcher and one creative, and  $C(q) = 5q$ . Each of them chooses how much to contribute to the quality of the model, taking the other person's contribution as given. What is the Nash equilibrium of this game? What is the socially optimal quality? Discuss the economic intuition of your results.

**3. (10 points) Nightclubs and Noise**

Berghain is one of the hottest nightclubs in Berlin. It also happens to be located directly next to a recording studio, Tresor.<sup>1</sup> Berghain produces an experience  $e$ , which it sells to consumers for a price of €40. To produce this experience, it also produces units of noise  $n$ . Berghain's cost function is  $C_B(e, n) = 10e + e^2 - 16n + n^2$ . Tresor produces audio  $a$ , which it sells to consumers at a price of €15. Tresor's cost function is  $C_T(a, n) = a + a^2 + an$ .

- (a) (2 points) Solve for the equilibrium quantities of  $e$  and  $a$ , and the noise level  $n$ . Assume that the firms make their decisions independently and without communicating.
- (b) (2 points) Suppose the two firms merge. What are the new equilibrium quantities of  $e$  and  $a$ , and the noise level  $n$ ? How does this compare to the previous equilibrium? Discuss the economic intuition of these results.
- (c) (2 points) What Pigouvian tax would the government need to impose to achieve the socially optimal level of noise?
- (d) (4 points) Discuss what kind of Coasean bargaining could be done to achieve the socially optimal level of noise under each of the following scenarios:
  - i. (2 points) Berlin allows unlimited noise pollution.
  - ii. (2 points) Anyone who can hear the noise can sue the nightclub for full compensation.

You do not need to solve for new equilibrium quantities, but you should discuss how the bargaining would proceed and the efficiency of the outcome.

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<sup>1</sup>Tresor is also a nightclub, for let's focus on its recording output

4. (5 points) **Brave New World**

Mustapha can employ two kinds of workers, alphas and betas.<sup>2</sup> An alpha can produce \$100 worth of output per day working for himself. If he works for Mustapha, he produces \$120 worth of output a day. A beta produces \$60 worth of output per day working for himself, and he produces \$80 worth of output per day if he works for Mustapha. Workers either work for themselves or work for Mustapha. Mustapha cannot tell alphas from betas. He pays a wage equal to the average product of his labor force and he has at least some alphas working for him. Workers are free to choose to work for themselves or for Mustapha, depending on which offers more money. Find the minimum percentage of Mustapha's workers that must be alphas in order for him to hire any alphas at all.

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<sup>2</sup>He will eventually diversify into gammas, deltas, and epsilons, but that's a story for another time.

## 5. Ungraded Questions

- (a) Barbie and Oppenheimer are two of the largest movie releases in the Summer of 2023. They also released on the same day. Suppose that AMC is the exclusive theater for Barbie, and Regal is the exclusive theater for Oppenheimer, and that the marginal cost of a ticket is 0. The demand for each movie is given by:

$$Q_B(p_B, p_O) = 200 - 2p_B + p_O$$

$$Q_O(p_B, p_O) = 100 - 2p_O + p_B$$

Note: Round equilibrium prices to the nearest dollar.

- i. Find the reaction functions for each theater, the equilibrium prices, and the number of tickets sold.
  - ii. Suppose that AMC and Regal merge. What are the new reaction functions, equilibrium prices and quantities?
  - iii. Suppose that the FTC blocks the merger discussed above, but allows AMC to establish a revenue-sharing agreement with Regal, wherein AMC receives 30% of Regal's revenue from Oppenheimer screenings. What are the new reaction functions, equilibrium prices and quantities?
  - iv. Compare these equilibrium outcomes. Which is best for consumers? Which is best for the theaters? Additionally, discuss the differences in the reaction functions in each equilibrium and how they give rise to the different outcomes.
  - v. There exists a subset of consumers for whom these films are complements, not substitutes: the Barbenheimer fans. Discuss how the pricing strategies of the theaters would change for Barbenheimer fans. Which of the above equilibrium outcomes would be best for Barbenheimer fans? Why? You do not need to solve for the equilibrium prices, but you should discuss how they would change in the context of reaction functions.
- (b) A group of 9 consumers are trying to decide whether to connect to a new communications network. Consumer 1 is of type 1, consumer 2 of type 2, consumer 3 of type 3, and so on. Each consumer's willingness to pay to belong to the network is proportional to the number of consumers who belong. Where  $k$  is the number of consumers who belong, the willingness to pay of a type  $n$  consumer is equal to  $k$  times  $n$ . What is the highest price at which 7 consumers could all connect to the network and either make a

profit or at least break even?